If the even number is greater than the odd one, we have $2m - 2n - 1 = 2(n - m) - 1$, the second form of an odd number.

*Prop.* VII.— The sum of any number of *even* numbers is even.

*Proof.*— Let $2r_1, 2r_2, 2r_3, \ldots 2r_n$ be any number of even numbers; then $2r_1 + 2r_2 + 2r_3 + \ldots + 2r_n = 2(r_1 + r_2 + r_3 + \ldots + r_n)$, which is even.

*Prop.* VIII.—The sum of an *even* number of odd numbers is even.

*Proof.*— Let $2r_1 + 1, 2r_2 + 1, 2r_3 + 1, \ldots 2r_{2n} + 1$ be any even number of odd numbers; their sum is $2r_1 + 2r_2 + 2r_3 + \ldots + 2r_{2n} + 2n = 2(r_1 + r_2 + r_3 + \ldots + r_{2n} + n)$ an even number.

*Prop.* IX.—The sum of an *odd* number of odd numbers is odd.

*Proof.*— $(2r_1 + 1) + (2r_2 + 1) + (2r_3 + 1) + \ldots + (2r_{2n+1} + 1) = 2(r_1 + r_2 + r_3 + \ldots + r_{2n+1}) + 1$, an odd number.

*Prop.* X.—The product of any number of even numbers is even.

*Proof.*— $(2r_1)(2r_2)(2r_3) \ldots (2r_n) = 2(2^{n-1}r_1r_2r_3 \ldots r_n)$.

*Prop.* XI.— The product of an odd number and an even number is even.

*Proof.*— $2r \times (2n + 1) = 2(2rn + r)$.

*Prop.* XII.— The product of any number of odd numbers is odd.

*Proof.*— $(2r_1 + 1)(2r_2 + 1)(2r_3 + 1) \ldots (2r_n + 1) = 2b + 1$, by putting $2b$ for all the terms of the product except 1.

Some years ago the following problem was proposed in a Medical Almanac: "It is required to put 20 horses in a stable containing 5 stalls, and have an odd number of horses in every stall."

Here we have an *even* number of horses and an *odd* number of stalls, and the sum of an *odd* number of odd numbers is required to be *even*, which by *Prop.* IX, is impossible.

Many persons spent hours of patient study in vainly endeavoring to solve this impossible problem.

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## *ON THE SOLUTION OF QUADRATIC EQUATIONS.*

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BY O. D. OATHOUT, READ, IOWA.

All complete quadratic equations can be made to take the form

(1) $$ax^2 + bx = c,$$

wherein $a$, $b$ and $c$ are coefficients in the most general sense of the term, and the highest exponent of $x$ is 2. Multiply (1) by $4a$ and add $b^2$ to each member and we have

(2) $4a^2x^2 + 4abx + b^2 = 4ac + b^2$. Extracting square root of (2) we get

(3) $$2ax + b = \sqrt{(4ac + b^2)}.$$

Now equation (3) can always be obtained directly from (1) without using (2) by observing the law of its formation, viz.; that the first member of (3) is the first derived polynomial of (1). Therefore all complete quadratic equations can be solved by the following rule:

1. Reduce to the form $ax^2 + bx = c$.
2. Make the first derived polynomial $= \pm\sqrt{(b^2 + 4ac)}$, using the double sign, and the roots of this equation are those required.

*Examles.* — 1. $3x^2 - 27x = -42$, to find the values of $x$.

The first derived polynomial is $6x - 27$; $\therefore 6x - 27 = \sqrt{(729 - 504)} = \pm 15$; $\therefore x = 7$ or $2$.

2. $3\sqrt[3]{x^2} - 10\sqrt[3]{x} = -3$, to find the values of $x$.

Put $\sqrt[3]{x} = y$; then $\sqrt[3]{x^2} = y^2$. Therefore the given equation becomes $3y^2 - 10y = -3$. First derived polynomial is $6y - 10$, $\therefore 6y - 10 = \sqrt{(100 - 36)} = \pm 8$; $\therefore y = 3$ or $\frac{1}{3}$; $\therefore x = 27$ or $\frac{1}{27}$.

3. $\frac{1}{3}x^2 + 5x = 42$, to find the value of $x$.

The first derived polynomial is $\frac{2}{3}x + 5$; $\therefore \frac{2}{3}x + 5 = \sqrt{(25 + 56)} = \pm 9$; $\therefore x = 6$ or $-21$.

It will be seen that if the first derived polynomial is made equal to 0, the resulting value of $x$ is that required for a maximum or a minimum.

The works of Davies, Robinson, Ray and Schuyler do not contain this method of solving quadratics. I discovered it last October; but it is probable that the method has long been known, though it may never have been published.

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## *SOLUTION OF A PROBLEM.*

BY THEO. L. DE LAND, WASHINGTON, D. C.

Supposing the sky, all the way from the zenith to the horizon, to be thickly dotted with stars, and that they are equally distributed in every part, what would be the mean of all their altitudes?

*Solution.* — Let $n$ be the number of stars, $h$ the altitude of any one of them, and $A$ its azimuth.

The area of an elementary portion of the sky is $\cos h.dhdA$ which must be taken as constant.

$$\text{Mean altitude} = \frac{h_1 + h_2 + h_3 + \ldots + h_n}{n}.$$